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CAN MASSLESS NEUTRINOS DOMINATE THE UNIVERSE?*

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ABSTRACT

The restrictions from cosmological considerations on masses and lifetimes of neutral, weakly-interacting fermions are reviewed. In particular, the possibility of the massless decay products of a heavy neutrino dominating the energy density of the present universe is discussed in detail.

INTRODUCTION

It has been over 15 years since Penzias and Wilson discovered the 3 K microwave background radiation.[1] The interpretation of this background as a remnant of the hot big bang is the cornerstone of modern theories of the beginning, the present, and the future evolution of the large scale structure of the universe. Despite the appearance of the clear night sky as viewed from the woods of northern Wisconsin, most of the photons in the universe do not originate in stars, but are present in the invisible 3 K background. Fifteen years of observation have confirmed the thermal nature of the background spectrum. A universe at a temperature of 3 K has about 400 photons per cubic centimeter, or about 10^{88} photons in the visible universe. This is a large number compared to the total number of neutrons and protons. There are only about 10^{80} nucleons in the universe: nucleons are only a small contaminant in a vast sea of photons. (Luckily, the nucleons are not uniformly distributed, as are the photons.) By observing the background photons, we directly probe the universe when the photons were last scattered. In the case of the background photons, the last scattering was when the universe was at a temperature of 10 K, or about 10^6 years after the big bang.

In addition to the background photons, there should also be a sea of neutrinos left over from the big bang, with about as many

of them as photons, about 10^{88}. Confirmation of this background
would be in some sense even more fundamental than the discovery
of Penzias and Wilson, since the background neutrinos last scattered
when the the temperature of the universe was 10^{10} K, or about
one second after the big bang. Thus, the background neutrinos
are an even older relic of the origin of the universe than the
background photons.

Although direct detection of the background neutrinos seems
remote, they may nevertheless play a crucial role in cosmology, and
even dominate the mass of the universe. Since the average energy
of a nucleon (rest-mass energy $\cong 10^9 eV$) is about 10^{12} times larger
than the average energy of a background photon (3 K $\cong 10^{-4}$ eV), in
determination of the mass-energy of the universe, the nucleons dom-
inate the more numerous (by a factor of 10^8) photons. However, if
there exist background neutrinos with energy greater than about
10eV, the larger mass of the nucleon would be compensated by the
sheer number of neutrinos. and the neutrinos would provide the
bulk of the energy density of the universe.

The future evolution of the universe is a fundamental question
in cosmology. Unfortunately, it is also an unanswered question.
Cosmological observations cannot yet determine if there is suf-
ficient gravitational energy in the universe to overwhelm the
expansion energy and cause an eventual recontraction, or if the
kinetic energy of expansion is greater than the potential energy,
and an infinite expansion will result.

There are three reasons to believe that the universe may be
closed. First, from the viewpoint of the theory of relativity, the
boundary condition for a closed surface is much more attractive
(i.e. simpler) than the boundary condition for an open (expansion
forever) universe. Second, Mach's principle, which guided Einstein
in the formulation of General Relativity, applies only to a closed
(eventual recontraction) universe.[2] The third reason is the "flat-
ness" problem which has been reviewed by Dicke and Peebles,[3] and
most recently studied by Guth.[4]

The flatness problem may be formulated as follows. Let ρ_0 be
the present energy density of the universe. If ρ_0 is greater than
a critical density, $\rho_C \equiv \dfrac{8\pi}{3} \dfrac{G}{H_0^2}$ where G is Newton's constant

and H is the present value of Hubble parameter, the universe will
be closed, and if $\rho < \rho_C$ the universe will be open. Observations
suggest that $10^{-2} \leq \rho_0/\rho_C \leq 10$.[5] This seems a large range, but
consider an earlier epoch. For conditions at the Planck temperature
(10^{19} GeV), the only timescale is the Planck time. For the universe
to survive to its present age ($t \cong 10^{18}$ sec $\cong 10^{50}$ Planck times) with

$\rho_0/\rho_C \sim 1$ requires a tuning of the Hubble parameter at the Planck time of about one part in 10^{60}. Stated succinctly, for the universe to have survived 10^{60} Planck times with $\rho_0 \cong \rho_C$ implies that at the Planck time, $\rho = \rho_C$ to one part in 10^{60}. (For a more precise statement of the problem, see ref. 4.) A solution to the problem that is somewhat less than completely arbitrary is to assume that $\rho = \rho_C$, i.e. $k = 0$ in the Robertson-Walker metric.

Although the three reasons given above are not conclusive evidence that $\rho_0 \geq \rho_C$, they nevertheless provide motivation to investigate the problem of whether the universe can be closed. Visible forms of matter seem to be incapable of closing the universe. The best observational evidence is that $\rho_{BARYONS} + \rho_{PHOTONS} \leq 10^{-1}\rho_C$.[6] Since there is no observational information about the primordial neutrinos, they are a likely candidate for the missing mass. It has long been known that primordial stable neutrinos with a mass of about 50eV can provide the missing mass to make $\rho \cong \rho_C$.[7] The purpose of this presentation is to demonstrate that this solution need not be unique, that massless neutrinos may today provide $\rho_0 = \rho_C$.

Below, I describe the decoupling of neutrinos in the early universe and limits on the mass and lifetimes of neutrinos as a result of observations of the present energy density. I also review other cosmological limits on neutrino lifetimes and discuss models for neutrino decay. Finally, I explore some observational consequences if the decay products of a heavy neutrino are responsibile for closing the universe.

PRIMORDIAL NEUTRINOS AND THE PRESENT ENERGY DENSITY

Observation of the Hubble expansion of the universe suggests that the universe was once in a hotter and denser phase. The thermal nature of the microwave background is evidence that the temperature of the universe was once high enough to ionize hydrogen, $T_U \geq 10eV$. Isolation of a primordial component in the universal helium and deuterium abundances implies that the temperature of the universe was once high enough for nucleosynthesis, $T_U \geq 1$ MeV. Observation of a global baryon asymmetry may be interpreted to require that the temperature of the universe was once large compared to the masses of particles mediating baryon number violating reactions, $T_U \geq 10^{12}$ GeV. It is necessary for us to assume only that the temperature of the universe was once greater than a few MeV.

Let ν_H be a "massive" neutrino, and ν_L be a "massless" neutrino. At sufficiently high temperatures, if the ν_H couple with the usual strength to the normal weak interaction bosons, they were

kept in thermal equilibrium through reactions such as $\nu_H\bar{\nu}_H \leftrightarrow \nu_L\bar{\nu}_L$. As the universe expanded and cooled, neutrino reactions became less frequent because expansion diluted the number density of neutrinos, and because the weak interaction cross section decreased as the energy of the neutrinos decreased. Finally the ν_H effectively decoupled, or froze-out, when the timescale for $\bar{\nu}_H$ interactions (G_F is the Fermi constant, and n_ν the neutrino number density),

$$\tau_I \equiv <n_\nu\sigma_\nu>^{-1}$$
$$\cong \left(G_F^2 T^5\right)^{-1} \quad , \tag{1}$$

became larger than the age of the universe (G is Newton's constant),

$$\tau_U^{-1} = G^{\frac{1}{2}}T^2 \quad . \tag{2}$$

The decoupling temperature, T_D, for weakly interacting fermions is found by equating (1) and (2):

$$T_D \cong 1 \text{ MeV} \quad . \tag{3}$$

For $T < T_D$, the neutrinos form a noninteracting gas and the total number of neutrinos is conserved. The number density of neutrinos is diluted only by the Hubble expansion. Since in an adiabatic expansion the total number of photons remains constant, a convenient parameter is the ratio of the number density of neutrinos and the number density of photons. Since for $T \leq T_D$ the number of neutrinos remains constant, n_ν/n_γ is roughly constant after decoupling, and

$$\frac{n_\nu}{n_\gamma} \cong \left.\frac{n_\nu}{n_\gamma}\right|_{T_D = 1 \text{ MeV}} \tag{4}$$

$$\cong \begin{cases} 1 & (m < 1 \text{ MeV}) \\ \left(\frac{m}{1 \text{ MeV}}\right)^{3/2} e^{-m/1\text{MeV}} & (m > 1 \text{ MeV}) \quad , \end{cases}$$

where the last equality follows from assuming the neutrinos were equilibrium distributed in phase space when they decoupled. A more exact result for n_ν is given by the solution to the transport equation:

$$\frac{dn_\nu}{dt} = - <\sigma|v|> (n_\nu^2 - n_\nu^{eq^2}) - 3 \frac{\dot{R}}{R} n_\nu \quad , \tag{5}$$

where n_ν^{eq} is the equilibrium number density, and $\dot{R}/R$ is the expansion rate of the universe.

If the ν_H survive, the present energy density of the neutrinos would be

$$\rho_o = mn_\nu$$

$$\simeq \begin{cases} mn_\gamma & (m < 1 \text{ MeV}) \\ mn_\gamma \left(\frac{m}{1\text{MeV}}\right)^{3/2} e^{-m/1\text{MeV}} & (m > 1 \text{ MeV}). \end{cases}$$

Although the energy density is not directly measurable (where does one put the scale to weigh the universe?) the present energy density may be expressed in terms of two measurable quantities, the Hubble parameter H_o, and the deacceleration parameter q_o:

$$\rho_o = 2q_o \left(\frac{3H_0^2}{8\pi G}\right) \quad . \tag{7}$$

The limit on ρ_o from the observational limits $q_o \leq 2$, $H_o \leq 100$ km s^{-1} Mpc^{-1}, implies that the neutrino mass must be less than about 50eV,[7] or greater than about 5 GeV.[8,9] More stringent limits on m may be found if additional assumptions are made about the contribution of neutrinos to the inferred galactic masses. However, the bound on ρ_o from H_o and q_o is the only reliable bound that is a result of <u>direct</u> observation.

The conclusion that no neutrino can have a mass in the range between 50eV and 5GeV may be easily circumvented if the neutrino decays to massless particles.[9,10] The crucial point is that in the expansion of the universe, the massive particles behave as a gas with an adiabatic index of 5/3, and massless particles behave as a gas with an adiabatic index of 4/3. Therefore, the energy density of massive (m > T) particles, ρ_H, decreases in expansion as T^3, while the energy density of massless (m < T) particles, ρ_R, decreases in expansion as T^4. Therefore, the contribution to the present energy density of the massless decay products is smaller than the contribution ν_H would make if it had <u>not</u> decayed by a

factor of $O(T_o/T_{Decay})$, where T_o is the present temperature, and T_{Decay} is the temperature of the universe at the time of ν_H decay. The ν_H lifetime as a function of mass that would result in the ν_H decay products contributing an energy density equal to the critical density is shown in Fig. 1.[9] The curve in Fig. 1 was found by calculating the number density and temperature at decoupling, n_D and T_D, as a function of the ν_H mass, and calculating the present energy density if ν decayed to massless particles with a lifetime τ:

$$\rho_o = mn(T_D)\left(\frac{To}{T_D}\right)^3 \int_{t_D}^{t_U} \left(\frac{t}{t_U}\right)^{1/2} \tau^{-1} \exp\left(-\frac{t - t_D}{\tau}\right) dt$$

Since observationally, $\rho_o \leq 2\rho_c$, the curve in Fig. 1 represents the minimum lifetime for any neutrino in the mass range $50\text{eV} \leq m_\nu \leq 5\text{GeV}$.

COSMOLOGICAL LIMITS ON ν_H LIFETIMES

The curve in Fig. 1 represents the minimum lifetime if the neutrinos decay to massless particles. There are additional cosmological limits on neutrino lifetimes:

(1) Lifetime Bound From the Solar Neutrino Experiment:[11] For large neutrino masses (2-5 GeV) the requirement that the present[12] neutrino background produced from ν_H decay not be detected in the Davis solar neutrino experiment places an upper bound on the ν_H lifetime. There is nothing to guarantee that the neutrinos detected by Davis are of solar origin. The bound is given as curve 1 in Fig. 3.

(2) Upper Lifetime Bound from Deuterium Abundance:[13] The major product manufactured by big-bang nucleosynthesis (at t =3 minutes when $\gamma + d \rightarrow p + n$ becomes negligible) is ^{4}He. The standard calculation of its abundance is in excellent agreement with observation ($\sim 26\%$ by weight). Deuterium is also believed to be of primordial origin, with a primordial abundance between 2×10^{-5} and 10^{-4} by weight. The one input parameter in nucleosynthesis calculations is h_o, the entropy per baryon at nucleosynthesis. If no entropy is generated in the universe between nucleosynthesis and the present time, h_o is related to the present baryon density by

$$\rho_B = 7.15 \times 10^{-27} h_o \quad . \tag{8}$$

In (8) ρ_B is the present baryon density ($\rho_B = \Omega_B \, 5.7 \times 10^{-30}$ gm
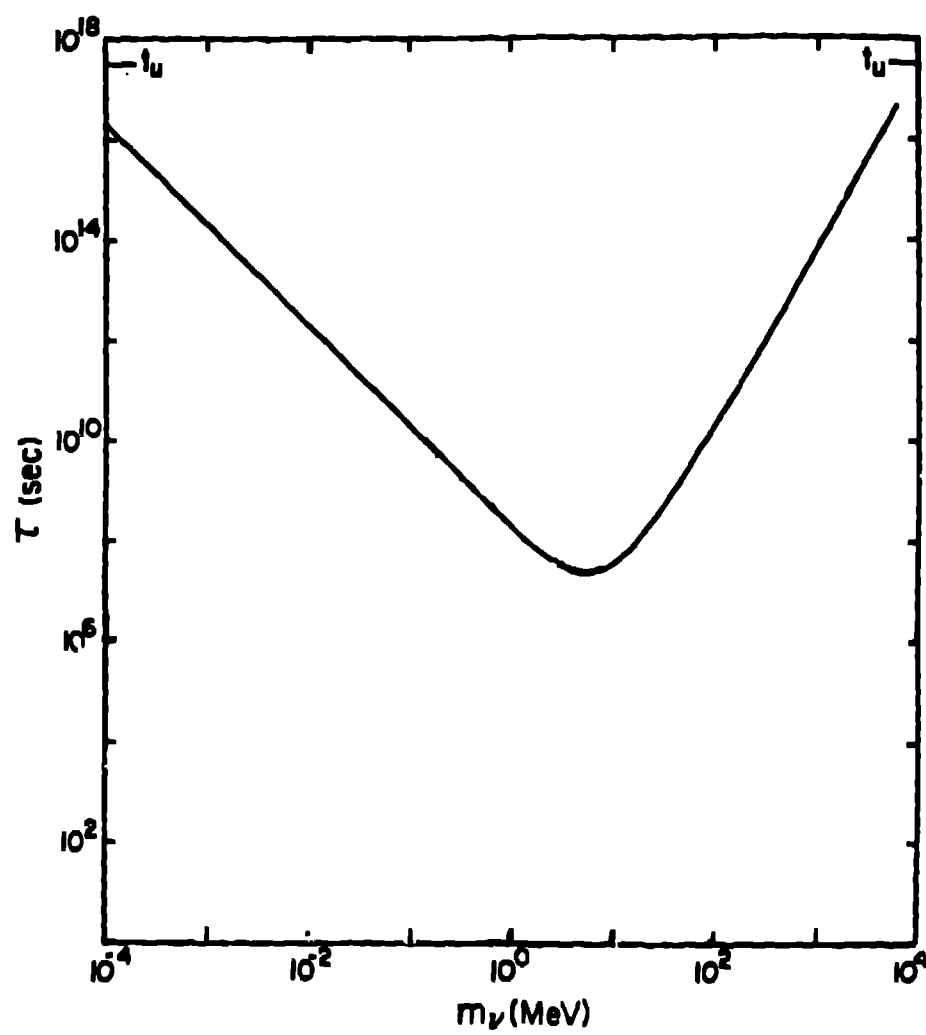
Fig. 1. The neutrino lifetime that results in the massless decay
products closing the universe.

cm^{-3}; $0.01 \leq \Omega_B < 1$. The present best determination of Ω_B is 0.06. The ^4He abundance is relatively insensitive to the input parameter, but the ^2H abundance is very sensitive to h_o. If $\nu_H \rightarrow \nu_L \gamma$ proceeds after the universe is dominated by the mass of the ν_H, it would greatly change the entropy per baryon. The temperature at which the universe is dominated by the ν_H mass is shown in Fig. 2. If ν_H decays after nucleosynthesis, and after the universe is dominated by its mass, Eq. (8) is no longer valid and should be replaced by

$$\rho_B = 2.65 \times 10^{-20} h_o /(T_1 x) \cdot \qquad (9)$$

In (9) T_1 is the temperature (in K's) at which ν_H dominates, and x is related to the lifetime, τ, for $\nu_H \rightarrow \nu_L \gamma$

$$\tau = (2.25 \times 10^7 \text{ sec}) x^2 \cdot \qquad (10)$$

Since limits are known on ρ_B and T_1 is known, Eq. (4) results in a limit on x. The limit on x results in a limit on τ from Eq. (10). An example of this limit for $\Omega = .06$ is given as line three in Fig. 3.

(3) Lifetime Bound From Thermalization of Photons:[13],[14] If the ν_H decays into a photon plus a massless neutrino, or into charged particles, the resulting photons must be thermalized. A bound on the ν_H lifetime, τ, may be set from the requirement that the decay photons be made early enough to be able to thermalize by the present time. The key to the thermalization of the photons from the decay of a massive ν_H is the degradation of the few γ's of energy $m_\nu/2$ to many γ's with average energy $k_B T$. The production of new, soft γ's proceeds through one of two standard paths: scattering, to excite an electron, followed by bremsstrahlung; or double Compton emission. The first process is especially sensitive to the baryon density. As a function of the present baryon density the cosmic lifetime must be less than $9 \times 10^6 \Omega_B^6$. For thermalization due to double Compton emission, the maximum ν_H lifetime allowed is $10^6 \Omega_B^{2/3}$. It is relatively insensitive to the precise value of the present baryon density. If we live in a low density universe as is currently believed, the double Compton process obtains, and the cosmic lifetime must be less than about 10^6 sec.* This is shown as curve 4 in Fig. 3.

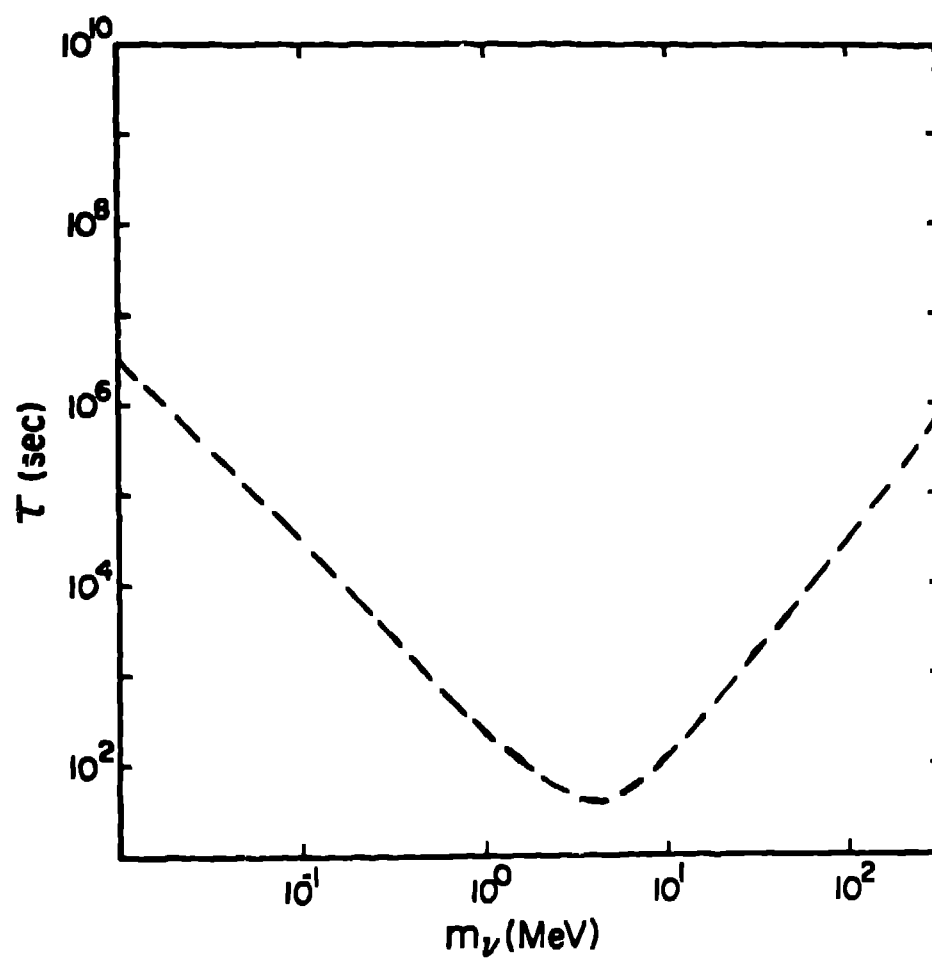*In Reference 14, a different thermalization bound is reported because this possibility was not included.

Fig. 2. The time of matter domination by ν_H.

(4) <u>Lower Lifetime Bounds From the Laboratory:</u>[13] The existence of the decay $\nu_H \to \nu_L + \gamma$ ($\nu_L = \nu_e$, ν_μ) implies that the production reaction $\nu_L + e \to \nu_H + e$ can proceed through γ exchange. Since the final state neutrino is never detected, the observed cross section for ν_L neutral current events provides an upper limit for ν_H production, hence an upper limit for the $\nu_H \nu_L \gamma$ effective coupling constant, and then, finally a <u>lower</u> limit for the lifetime for the decay $\nu_H \to \nu_L + \gamma$. In figure 3 we show the minimum lifetime from the Reines experiment to measure $\nu_e e$ scattering.[15] The constraint is given as line 5.

The lifetime bounds (2) and (3) are the best upper bounds. Comparison of the cosmological upper bounds and the experimental lower bounds implies that no neutrino with a radiative decay can exist if m $\lesssim$ 0.1 MeV. It may also be noticed that if $\nu_H \to$ entropy is the major decay mode, the restrictions given above forbid the ν_H lifetime to be long enough for its decay products close the universe.

MODELS FOR ν_H DECAY

If neutrinos are massive, in the absence of a global symmetry the heaviest neutrino will decay to the lighter ones. There are several models that may be employed to estimate the neutrino lifetime. Several of these models were considered by Goldman and Stephenson.[10]

First consider estimates for $\nu_H \to \nu_L \gamma$. The only gauge invariant form for the matrix element is

$$| M | = f\bar{u}(p') \sigma^{\mu\nu} q_\nu (1 \pm \gamma_5) u(p) \varepsilon_\mu, \qquad (11)$$

where $p = p' + q$, ε_μ is the polarization vector for the photon and f is an arbitrary coupling constant of dimension mass^{-1}. Consider three possibilities for f: (A) the result of first-order weak with neutrino mixing, (B) first-order weak with heavy charged leptons, and (C) GIM suppressed second-order weak (actually order G_F/M_W^2). The $\nu_H \to \nu_L \gamma$ lifetimes in the three cases above are shown by the three bands in Fig. 3 for reasonable choices of the parameters. For more details on the models, see Refs. 10 and 13.

Of particular interest is a model[16] in which the main decay mode does not create entropy, so that the bounds discussed above need not apply and there is a possibility of the ν_H decay products closing the universe. Assume that in addition to the known lepton doublets there is a neutrino singlet which mixes with the neutrinos in the doublets. The lifetime for $\nu_H \to \nu_L \bar{\nu}_L \nu_L$ would be (β is the singlet-doublet mixing angle)
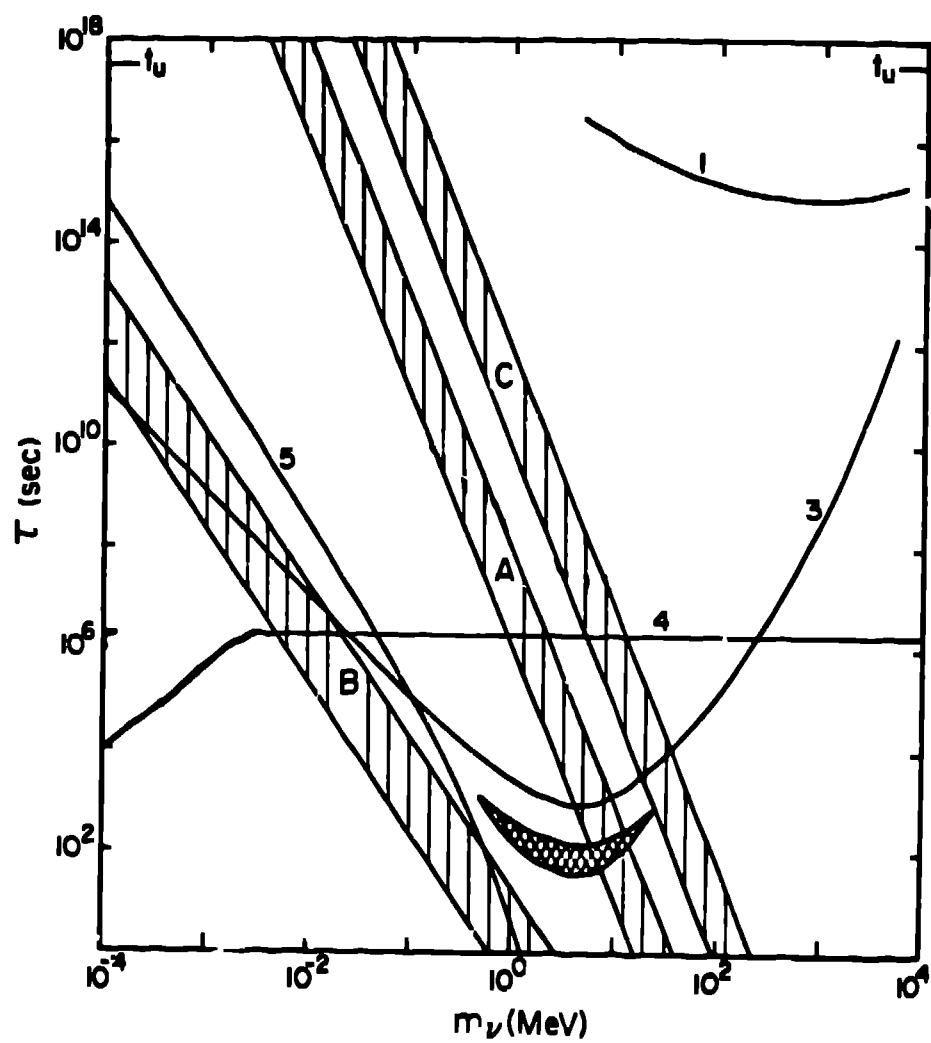
Fig. 3. Cosmological upper and experimental lower bounds on the lifetime for $\nu_H \rightarrow \nu_L \gamma$.

$$\tau(\nu_H \rightarrow \nu_L\nu_L\nu_L) = \left[\frac{G_F^2 \; m^5 \sin^2 2\beta}{192\pi^2}\right]^{-1} \quad (12)$$

$$\simeq 3 \times 10^4 \sin^{-2} 2\beta \left(\frac{1 \text{ MeV}}{m}\right)^5 \text{ sec.}$$

The lifetime for $\nu_H \rightarrow \nu_L\gamma$ is longer:

$$\tau(\nu_H \rightarrow \nu_L\gamma) = \left[\frac{25}{36} \frac{G_F^2 m^5 \; \alpha \sin^2 2\beta}{514\pi}\right]^{-1}$$

$$\simeq 6 \times 10^7 \sin^{-2} 2\beta \left(\frac{1\text{MeV}}{m}\right)^5 \text{ sec} \quad . \quad (13)$$

The lifetimes as a function of mass are shown in Fig. 4 if $10^{-4} \leq \sin^2 2\beta \leq 10^{-1}$. Also shown in Fig. 4 is the lifetime necessary if the ν_H decay products are to close to the universe. The relevance of this model is that if $0.1\text{MeV} \leq m \leq 1$ MeV, there is a possibility that the ν_H decay products close the universe since the bounds mentioned in the previous section apply only to entropy producing decays.

CAN MASSLESS NEUTRINOS CLOSE THE UNIVERSE?

Assume that a singlet neutrino exists in the mass range $0.1\text{MeV} \leq m \leq 1$ MeV with the requisite mixing angles for the lifetime to be the necessary value for the ν_H decay products to close the universe, 2×10^8 s $\leq \tau \leq 2 \times 10^{10}$ s. We now discuss further implications for this model.

(1) Primordial Decays: Although the main decay mode does not create photons, about 10^{-4} of the ν_H will create photons. Since we are assuming that the decay products close the universe, about 10^{-4}

of the closure density must be in photons. There are two possibilities; either the photons have simply redshifted and are today hidden in the far UV where the opaqueness of our galaxy at these wavelengths would prevent detection of the background photons, or the decay photons ionized the hydrogen, scattered with the electrons, and thermalized to form the present microwave background.[17] The latter possibility would explain why about 10^{-4} of the closure density is in the thermal background. Since only a relatively small number of photons are being produced, the thermalization
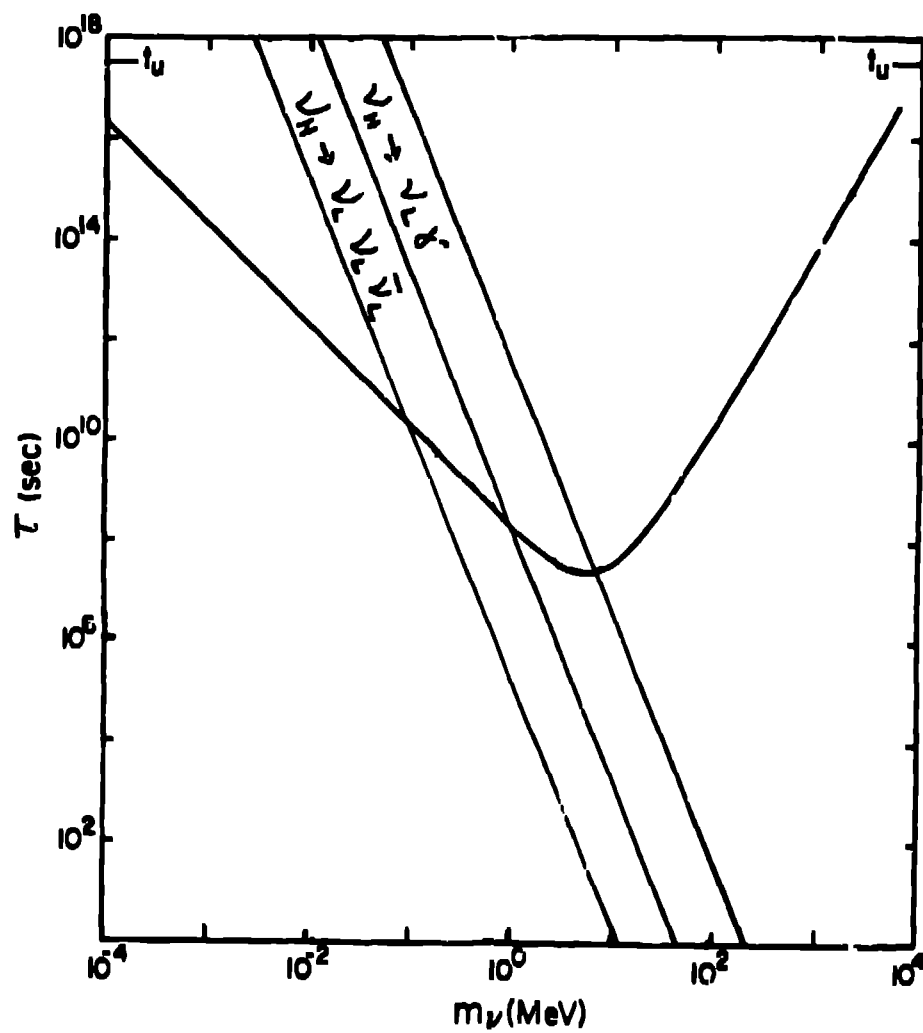
Fig. 4. Lifetimes for $\nu_H \to \nu_L \nu_L \nu_L$ and $\nu_H \to \nu_L \gamma$ in a model with an unpaired neutrino.

bounds discussed above are not applicable. In addition, if the "correct" number of photons are produced, it may be possible to achieve thermalization by Thomson scattering, which proceeds much faster than the thermalization process described above.

(2) Decays of ν_H Produced in Supernovae: Cowsik[18] has pointed out that since the bulk of the binding energy released in the formation of a neutron star is released in neutrinos,* if the neutrino decay produces photons, a background gamma-ray flux would result. If the $\nu_H \to \nu_L \gamma$ lifetime is less than the age of the universe, the gamma-ray flux from the decay products of the ν_H produced in supernovae would be

$$F_\gamma = \frac{GM^2}{RE_\nu} \quad \frac{\rho_B R_U}{M_{gal}} \Gamma_{SN} R_\gamma \quad , \tag{14}$$

where M and R is the mass and radius of the resultant neutron star, E_ν is the average energy of the emitted neutrino, ρ_B is the baryon density, M_{gal} is a typical galatic mass, R_U is the radius of the universe, $\Gamma_{S.N}$ is the supernovae frequency and R_γ is the fraction of ν_H that produces photons. Putting in reasonable values for the above parameters F_γ is about $10^{-3} - 10^{-4}$ cm^{-2} s^{-1} sr^{-1}, which is just below the allowed limits.[19]

The conclusions are: If neutrinos exist with masses in the range 50eV $\leq$ m $<$ 5GeV, they must be unstable. If the decay of the neutrino produces entropy, there are good limits on the possible lifetimes, and neutrinos with masses less than 10^{-1} MeV are forbidden. There is a reasonable model where a non-entropy producing decay dominates. In this model the neutrino decays predominately to three light neutrinos, and only about 10^{-4} of the initial neutrinos produce photons. The decay products in this model can dominate the universe if the neutrino is in the mass range 0.1MeV $\leq$ m $\leq$ 1 MeV, which results in a neutrino lifetime 10^8 s $\leq$ τ $\leq$ 10^{10} s. If this model is viable, 10^{-4} of the critical density must today be in photons, either thermalized in the microwave or hidden in the far UV. The decay of the neutrinos produced in supernovae could account for the observed gamma-ray flux.

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*This assumption may seem somewhat unfounded since we cannot even predict correctly the neutrino flux from our sun.

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